

# Hierarchical Graph Representation Learning with Differentiable Pooling

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**Speaker:** Ziyuan Ye

Thursday, August 18, 2022

# About the authors



Zhitao (Rex) Ying

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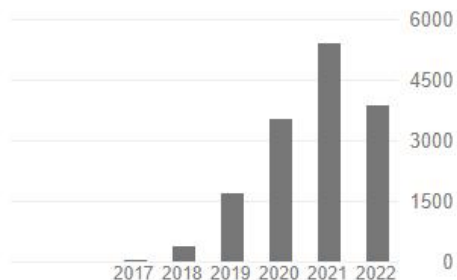
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Hierarchical graph representation learning with differentiable pooling Z Ying, J You, C Morris, X Ren, W Hamilton, J Leskovec Advances in neural information processing systems 31	873	2018
Graph convolutional policy network for goal-directed molecular graph generation J You, B Liu, Z Ying, V Pande, J Leskovec Advances in neural information processing systems 31	600	2018

## Representative Work:

- ✓ GraphSAGE
- ✓ GNNExplainer
- ✓ DIFFPOOL

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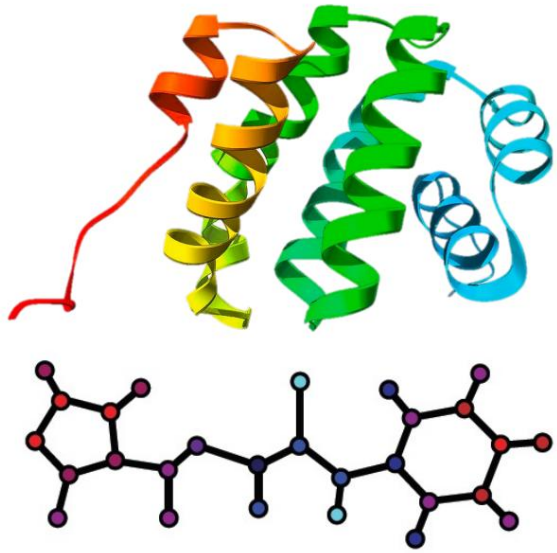
# Content

1. Background
2. Research content
3. Experimental results
4. Take-home message

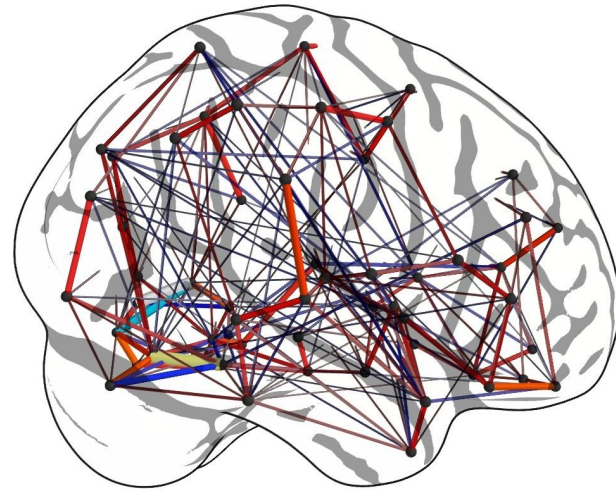
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# Graphs = System of Relations and Interactions



Molecules

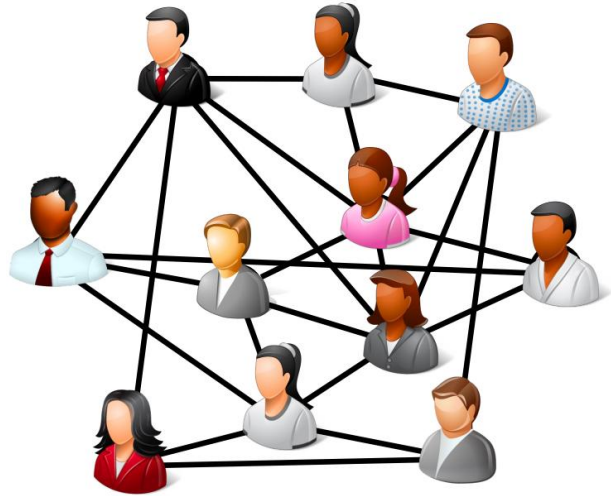


Brain connectivity

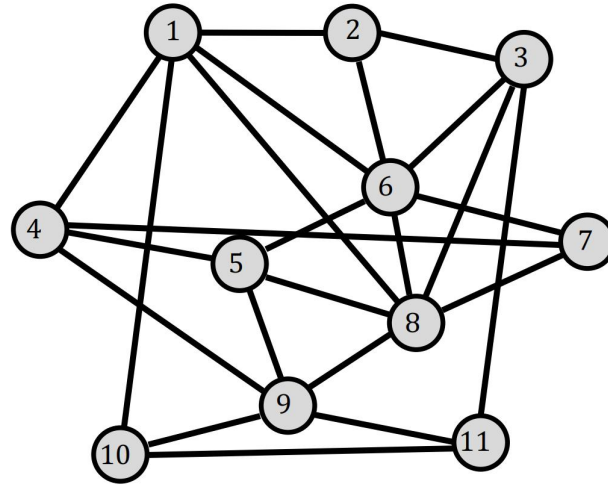


Social Network

# Graph Classification via Graph Neural Networks (GNNs)

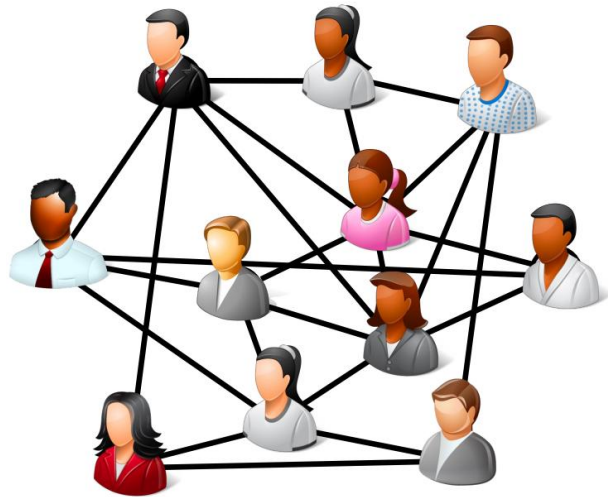


**Social network**

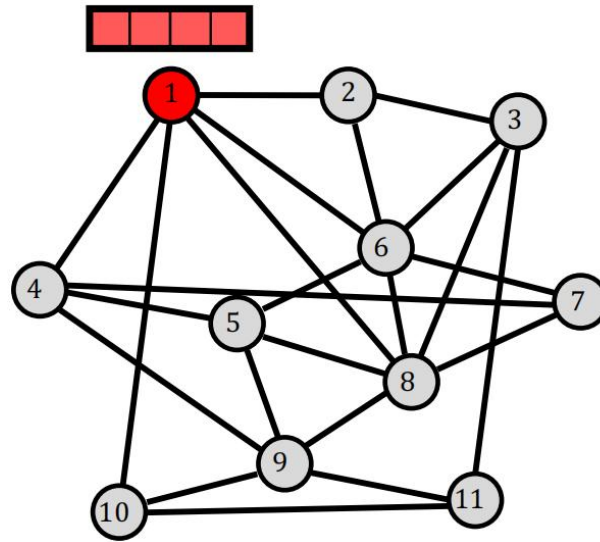


**Graph**

# Graph Classification via GNNs

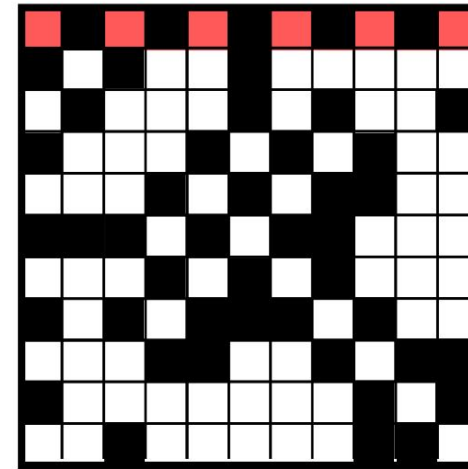


Social network



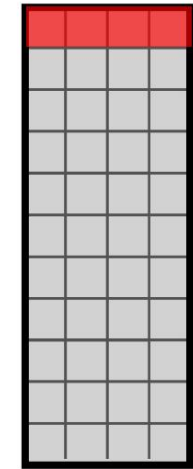
Graph

Adjacency  
matrix  $n \times n$



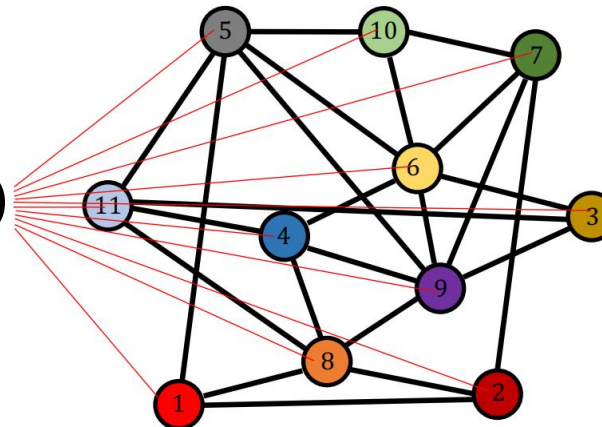
A

Feature  
matrix  $n \times d$



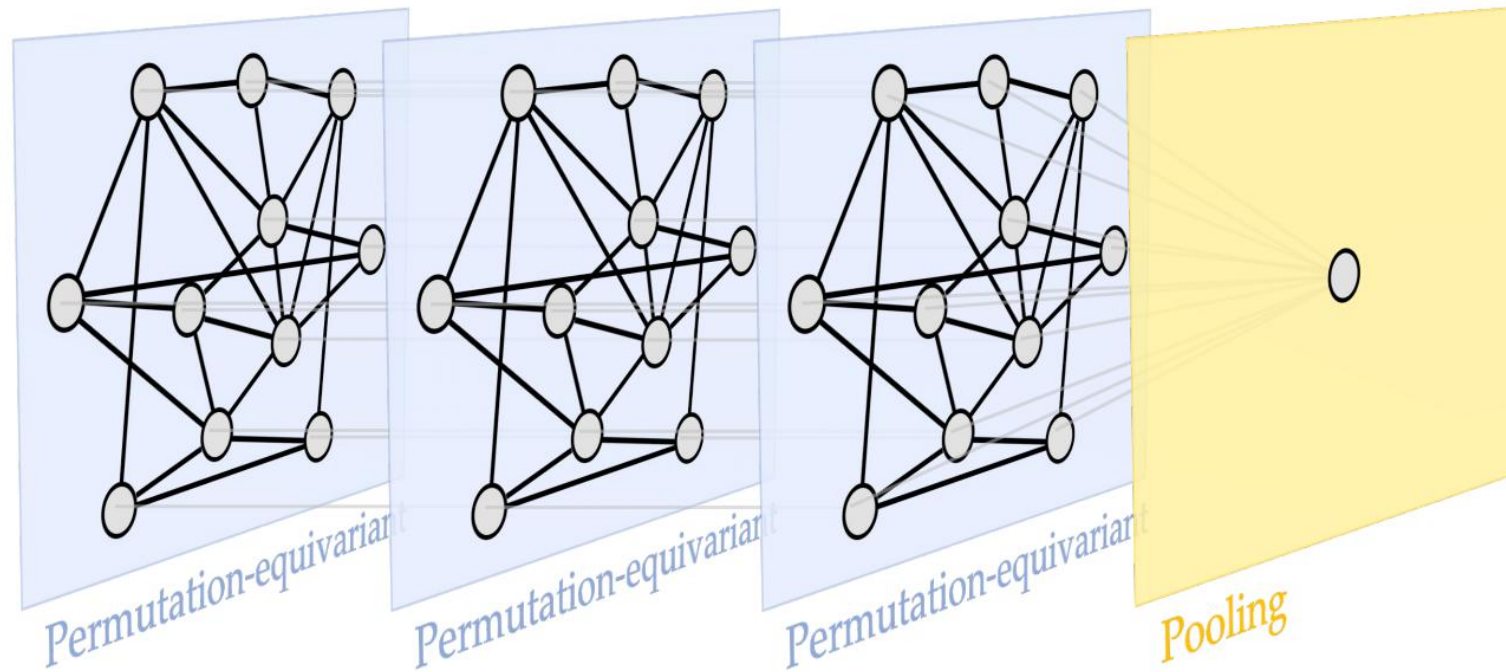
X

Graph function  $f(X, A)$





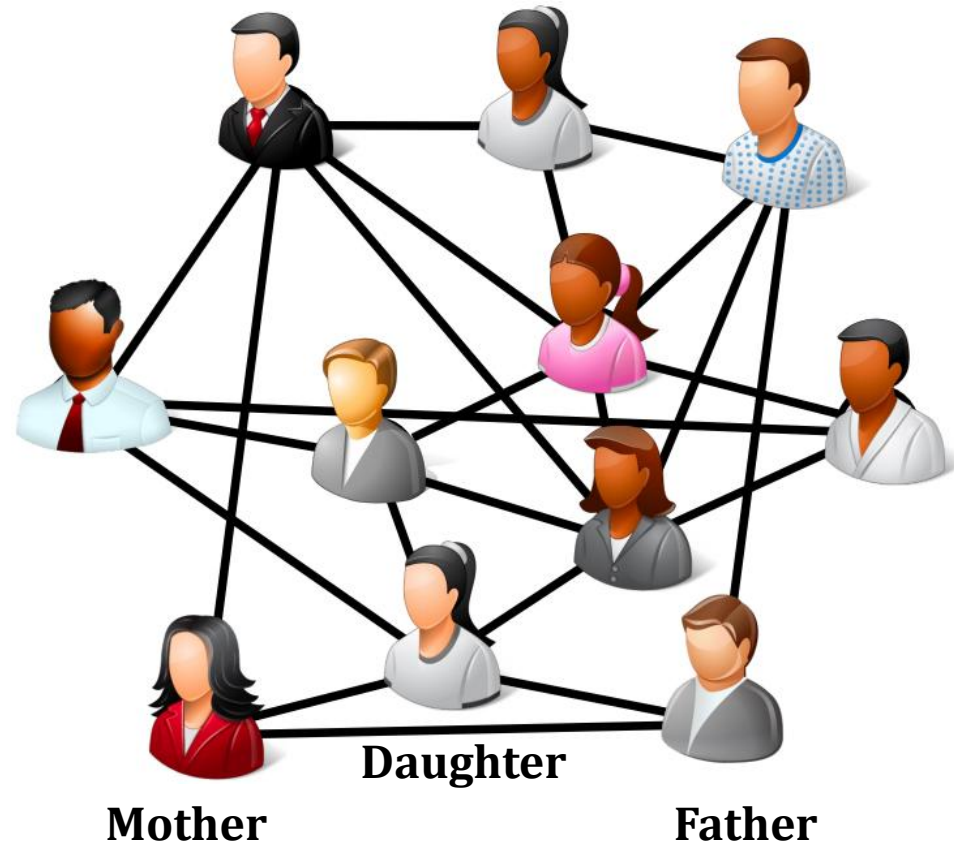
# Graph pooling in GNNs



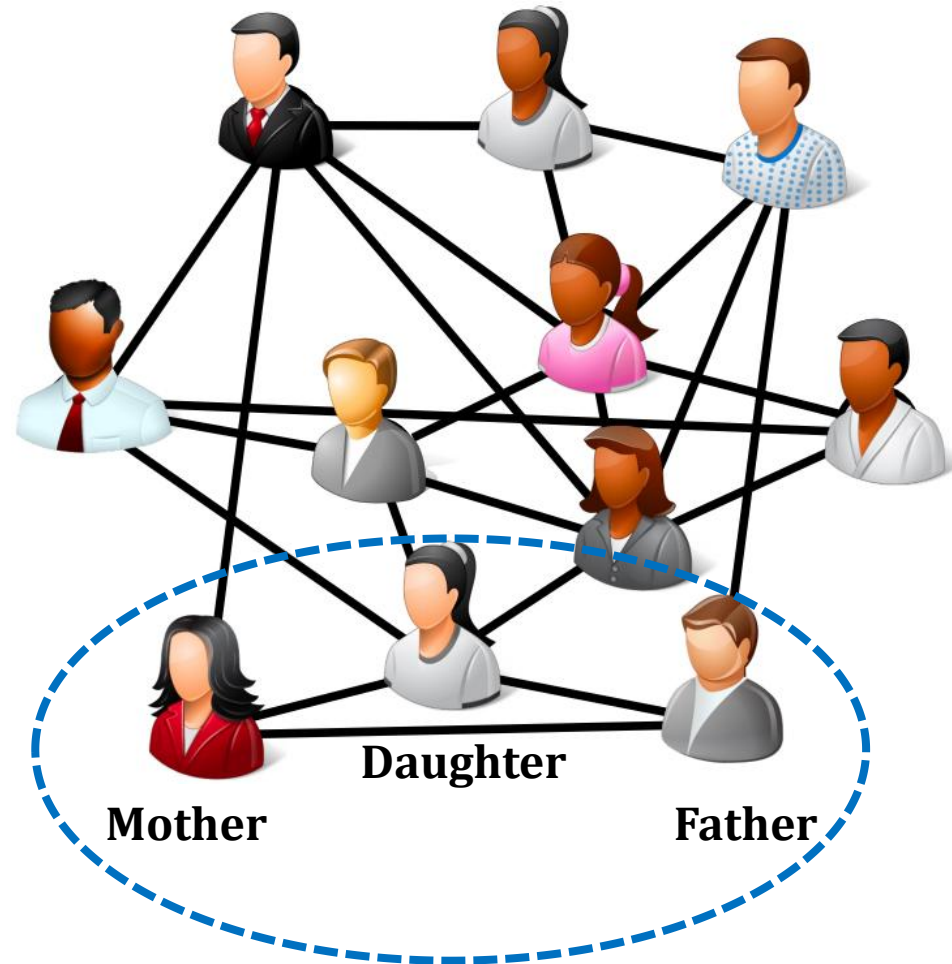
*Ignoring any hierarchical structure* that might be present in the graph



# Graph pooling in GNNs



# Graph pooling in GNNs



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# Notation

**Adjacency matrix**

$$A \in \{0, 1\}^{n \times n}$$

**Feature matrix**

$$F \in \mathbb{R}^{n \times d}$$

**Graph**

$$G \text{ as } (A, F)$$

**Node embeddings  
computed after  $k$   
steps of GNN**

$$H^{(k)} \in \mathbb{R}^{n \times d}$$

**Node embeddings  
of coarsened graph**

$$Z^{(k)} \in \mathbb{R}^{m \times d}, m < n$$

**Dataset**

$$\mathcal{D} = \{(G_1, y_1), (G_2, y_2), \dots\}, \\ y_i \in \mathcal{Y}$$

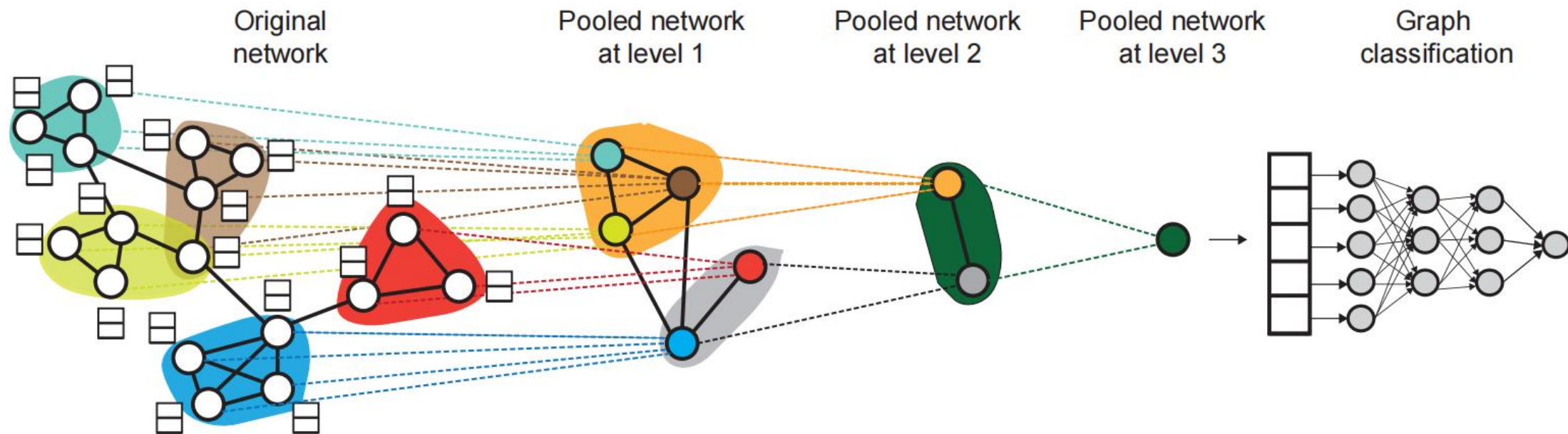
**Learnable cluster  
assignment matrix**

$$S^{(l)} \in \mathbb{R}^{n_l \times n_{l+1}}$$

**Goal of graph  
classification**

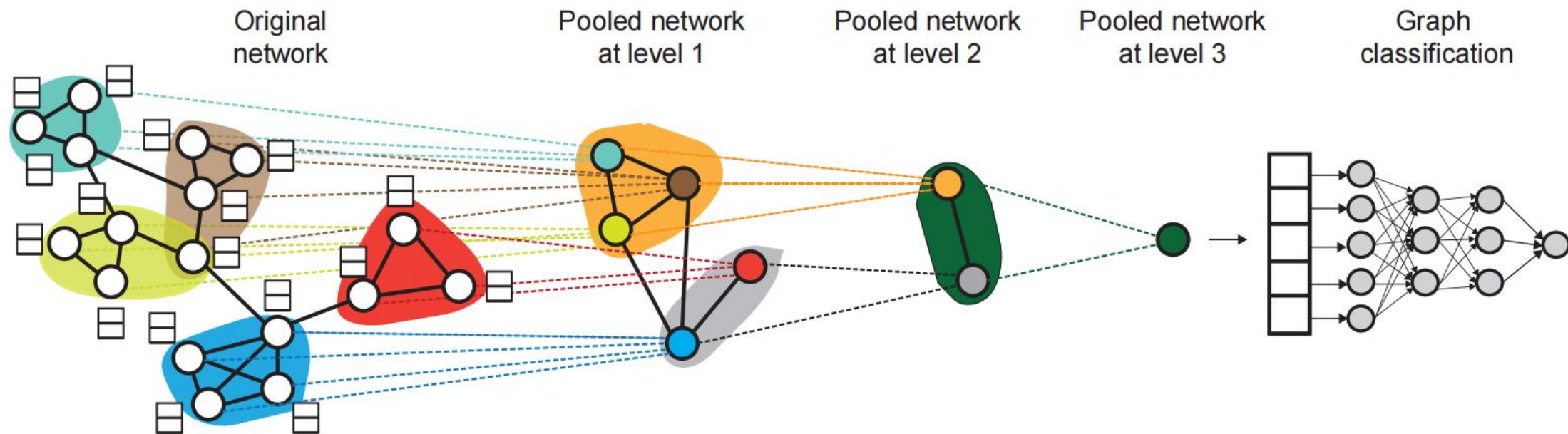
$$f : \mathcal{G} \rightarrow \mathcal{Y}$$

# DIFFPOOL



Graph **coarsening** and **classification** via DIFFPOOL

# DIFFPOOL



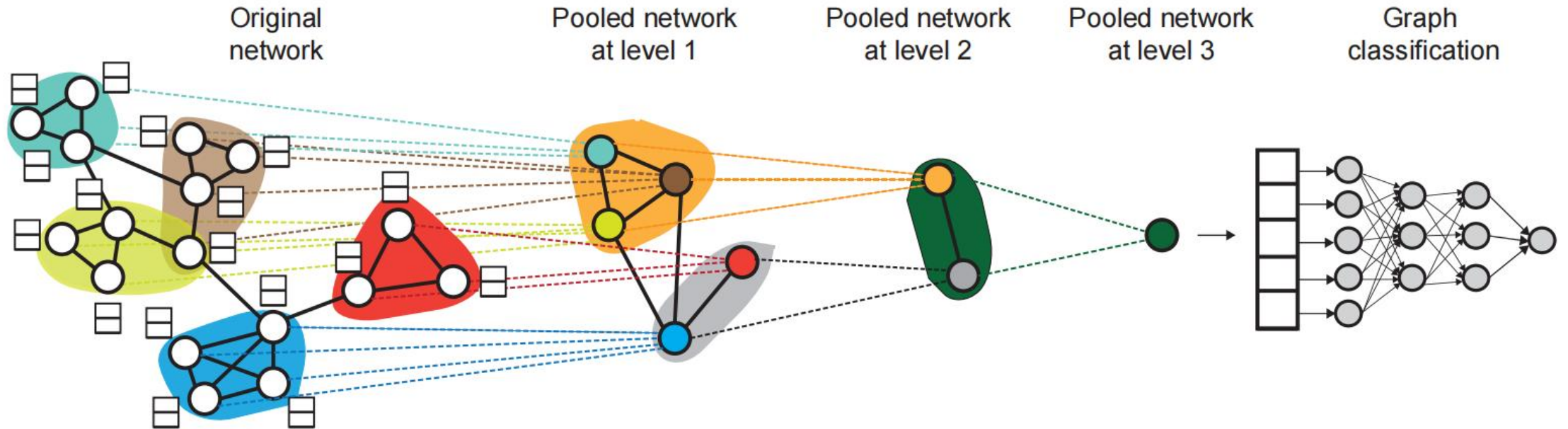
Graph **coarsening** and **classification** via DIFFPOOL

**Learning cluster assignment matrix**

$$S^{(l)} = \text{softmax} \left( \text{GNN}_{l, \text{pool}}(A^{(l)}, X^{(l)}) \right)$$



# DIFFPOOL



Graph **coarsening** and **classification** via DIFFPOOL

**Learning cluster assignment matrix**

$$S^{(l)} = \text{softmax} \left( \text{GNN}_{l,\text{pool}}(A^{(l)}, X^{(l)}) \right)$$

**Updating embeddings and adjacency matrix**

$$Z^{(l)} = \text{GNN}_{l,\text{embed}}(A^{(l)}, X^{(l)})$$

$$X^{(l+1)} = S^{(l)T} Z^{(l)} \in \mathbb{R}^{n_{l+1} \times d},$$

$$A^{(l+1)} = S^{(l)T} A^{(l)} S^{(l)} \in \mathbb{R}^{n_{l+1} \times n_{l+1}}$$

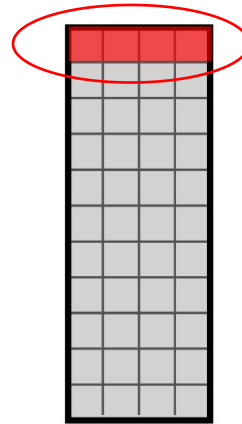


# DIFFPOOL: Regularization terms

## Entropy Regularization

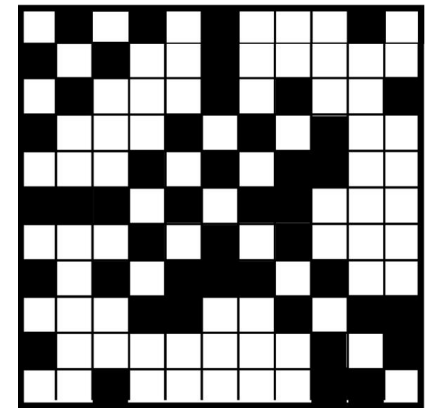
$$L_E = \frac{1}{n} \sum_{i=1}^n H(S_i)$$

Assignment matrix  $n \times m$



With: [0.95, 0.01, 0.02, 0.02]

Without: [0.33, 0.34, 0.20, 0.13]



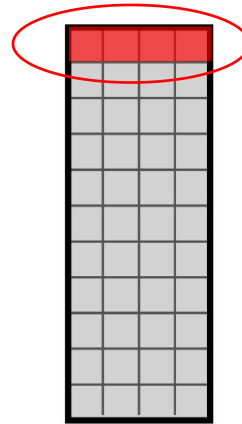
Adjacency  
matrix  $n \times n$

# DIFFPOOL: Regularization terms

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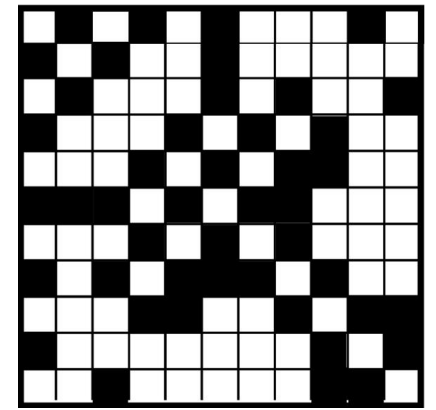
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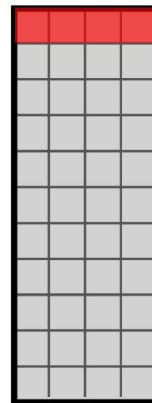


Adjacency matrix  $n \times n$

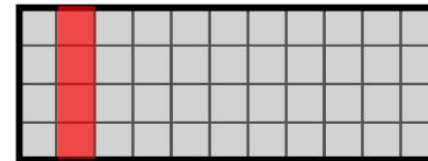
## Auxiliary Link Prediction Objective

$$L_{LP} = \|A^{(l)}, S^{(l)} S^{(l)T}\|_F$$

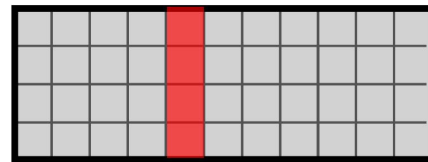
$S^{(l)}$



$S^{(l)T}$



The higher the better



The lower the better

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# Experimental results

	Method	Data Set					Gain
		ENZYMES	D&D	REDDIT-MULTI-12K	COLLAB	PROTEINS	
Kernel	GRAPHLET	41.03	74.85	21.73	64.66	72.91	
	SHORTEST-PATH	42.32	78.86	36.93	59.10	76.43	
	1-WL	53.43	74.02	39.03	78.61	73.76	
	WL-OA	60.13	79.04	44.38	80.74	75.26	
GNN	PATCHYSAN	–	76.27	41.32	72.60	75.00	4.17
	GRAPHSAGE	54.25	75.42	42.24	68.25	70.48	–
	ECC	53.50	74.10	41.73	67.79	72.65	0.11
	SET2SET	60.15	78.12	43.49	71.75	74.29	3.32
	SORTPOOL	57.12	79.37	41.82	73.76	75.54	3.39
	DIFFPOOL-DET	58.33	75.47	46.18	<b>82.13</b>	75.62	5.42
	DIFFPOOL-NO LP	61.95	79.98	46.65	75.58	76.22	5.95
	DIFFPOOL	<b>62.53</b>	<b>80.64</b>	<b>47.08</b>	75.48	<b>76.25</b>	<b>6.27</b>

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# Take-home Message

- **Motivation:**

- Original GNN architectures are unable to infer and aggregate the information in a hierarchical way.

- **Challenge:**

- Graphs contain no natural notion of spatial locality, i.e., one cannot simply pool together all nodes in a “ $m \times m$  patch” on a graph.

- **Main contributions:**

- Propose DIFFPOOL that can generate hierarchical representations of graphs;
- DIFFPOOL can be combined with various graph neural network architectures in an end-to-end fashion;

Thanks for your attention!